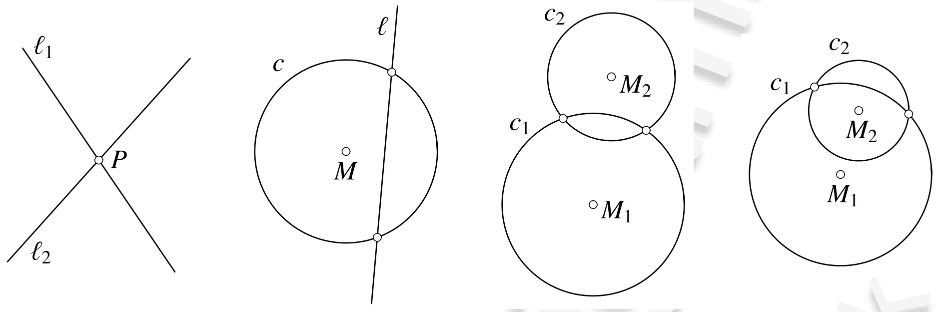

(E3) (E4) (E5) *(i)* (E5) *(ii)*

— *Figure 1.2* —

Iterated applications of (E1) to (E5) lead from a priori knowledge of specific points to specific straight lines and circles, then to further points, and so on. (Note that the tangent cases — line tangent to a circle or two touching circles — are usually assumed to require constructions in order to precisely determine the points of tangency. These cases are assumed to be included in (E4) and (E5).) A geometric construction problem is said to be "solvable" by Euclidean methods, if it can be shown that iterated application exclusively of (E1) to (E5) leads from certain given points to those points and/or straight lines and/or circles that have whatever geometric property is required.

3 Elementary geometric procedures of origami

Origami is, of course, the art of paper folding. (*Kami* is the Japanese word for paper, and *ori* the Japanese for folding; thus ori-kami, or origami.) As anyone who has ever put a crease in a piece of paper knows, there are certain procedures in paper folding that seem natural and basic. It is natural to fold a straight line, for instance, whereas folding a curve is possible, but difficult to control. (Although there are models in practical origami utilizing curved folds, for the moment we shall exclude these as non-elementary.) Folding a plane sheet of paper in such a way that a crease in the form of a line segment is created can be said to correspond to dividing a plane into two half-planes by a straight line and rotating one half-plane onto the other with the delineating line as the axis of rotation. It is very difficult to imagine a similar interpretation of folding curved creases, and in fact it requires concepts from calculus to do so. In this book, we shall restrict our considerations to straight folds exclusively.

As an origami model develops, its increasing complexity creates an increasingly complex analogous geometric pattern composed of straight lines (or, more precisely, line segments) on the paper used in producing the model. In most origami models, the result is in some way three-dimensional (although folding 2-dimensional forms such as regular polygons or stars is also sometimes considered), but every model can be opened up, and what we are considering in this book is essentially the geometry of the folds on the opened paper. Some abstraction is necessary, of course. Although most origami is done with squares (or