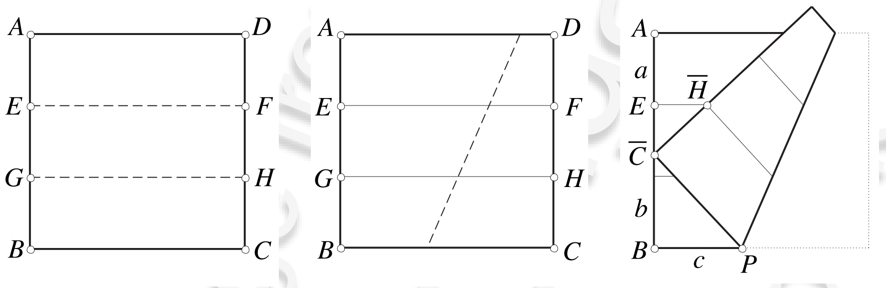
to solve by Euclidean or origami methods), and it took hundreds of years of research to finally prove the impossibility of such a construction.

A very elegant solution to this problem using origami methods was proposed by Peter Messer in [50] (Figure 2.9).



(a) (b) (c)

— *Figure 2.9* —

First of all, in Figure 2.9a a square is folded into three equal sections, such that

$$AE = EG = GB = DF = FH = HC.$$

(Methods of doing this are presented in section 12 of Chapter 3, page 53)

In Figures 2.9b and 2.9c, the square is folded such that C comes to lie on a point $\overline{C}$ on AB, and H simultaneously comes to lie on a point $\overline{H}$ on EF (by virtue of (O7*)). It then turns out that the length of $A\overline{C}$ is $\sqrt[3]{2}$ times the length of $\overline{C}B$.

Proof If we let a denote the length of $A\overline{C}$ and b denote the length of $\overline{C}B$, the folding square has sides of length $a + b$. We use P to denote the end-point of the crease created in Figure 2.9b on BC, and c to denote the length of BP.

Since $$P\overline{C} = PC,$$

we have $$P\overline{C} = (a + b) - c,$$

and since $\triangle BP\overline{C}$ is a right triangle, we have

$$b^2 + c^2 = (a + b - c)^2$$

or $$0 = a^2 + 2ab - 2c(a + b),$$

and therefore $$c = \frac{a^2 + 2ab}{2(a + b)}.$$

Triangles $\triangle BP\overline{C}$ and $\triangle E\overline{C}\,\overline{H}$ are similar, since

$$\angle\overline{C}BP = \angle\overline{H}E\overline{C} = 90^\circ$$

and

$$\begin{aligned}\angle\overline{C}PB &= 180^\circ - \angle\overline{C}BP - \angle B\overline{C}P \\ &= 90^\circ - \angle B\overline{C}P\end{aligned}$$