

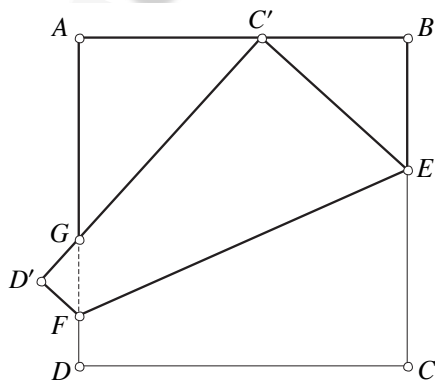
### 13 Six problems from one fold

In this section we consider six intimately related problems, which all result from folding a square of paper once. These problems demonstrate the power of the Haga Theorem in all its nuances.

I suggest that readers of this section attempt to solve the problems themselves before they read the solutions so that they gain greater familiarity with the situation. In order to make this easier, I will state the problems first, and save the solutions for the end of the section.

The situation we are given in all of the problems is the following (Figure 3.10).

Let  $ABCD$  be a square of paper placed on a plane surface. Holding corners  $A$  and  $B$  on the surface (and identifying the points immediately below  $C$  and  $D$  in the surface with  $C$  and  $D$  themselves),  $C$  is folded onto the point  $C'$  on  $AB$ . Simultaneously,  $D$  is folded onto a point  $D'$ . Folding  $C$  onto  $C'$  in this way creates a crease  $EF$  in the paper with  $E$  on  $BC$  and  $F$  on  $DA$ . Furthermore, we let  $G$  designate the point of intersection of  $DA$  and  $C'D'$ .



— Figure 3.10 —

#### Problem<sup>a</sup> 1

Prove that  $C'D'$  is a tangent of the circle with center in  $C$ , passing through  $B$  and  $D$ .

#### Problem<sup>b</sup> 2

Prove that the perimeter of triangle  $GAC'$  is equal to half the perimeter of  $ABCD$ .

#### Problem 3

Prove the identity  $AG = C'B + GD'$ .

#### Problem<sup>c</sup> 4

Prove that the sum of the perimeters of triangles  $C'BE$  and  $GD'F$  is equal to the perimeter of triangle  $GAC'$ .

#### Problem 5

Prove that the perimeter of triangle  $GD'F$  is equal to the length of line segment  $AC'$ .

#### Problem<sup>d</sup> 6

Prove that the inradius of  $GAC'$  is equal to the length of line segment  $GD'$ .

<sup>a</sup>Posed on p 11 of [29].

<sup>b</sup>Problem Number 2 in the VIII Nordic Mathematical Contest, 1994.

<sup>c</sup>Problem Number 4, Grade Two, Final Round in the 37th Slovenian Mathematical Olympiad, 1993.

<sup>d</sup>Example 3.1 on page 37 of [10].