We therefore have

$$\cos 36^\circ = \frac{\phi}{2} = \frac{\sqrt{5}+1}{4},$$

and since

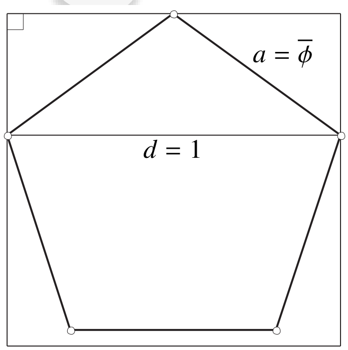
$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}},$$

it follows that

$$\begin{aligned}\sin 18^\circ &= \sqrt{\frac{1-\frac{\sqrt{5}+1}{4}}{2}}\\ &= \frac{\sqrt{5}-1}{4}\\ &= \frac{\bar{\phi}}{2}.\end{aligned}$$

20 Some precise methods for folding a regular pentagon

With all of the above at our disposal, we now turn our attention to actually folding regular pentagons. A first, relatively simple method of folding a regular pentagon can be derived from the fact that the length of the diagonal is ϕ times the length of the side. We assume that the folding square has unit edge length, and that the pentagon is inscribed in the square such that a diagonal is parallel to a side of the folding square, and one vertex of the pentagon is the mid-point of a side of the square (Figure 6.4).



— Figure 6.4 —

The sides of the pentagon then have length

$$\frac{1}{\phi} = \bar{\phi} = \frac{\sqrt{5}-1}{2}.$$

Application of this fact yields the following model.