# Chapter 8

# A Few More Polygons

## 25 The regular nonagon

In the last two sections, we mentioned that being able to fold a regular $n$-gon means that it is also possible, at least in theory, to fold any regular $(n \cdot 2^k \cdot 3^l)$-gon. Since the angle subtended over a side at the center of a regular $n$-gon is equal to $\alpha_n = \frac{360^\circ}{n}$, the angle subtended over a side at the center of a regular $(n \cdot 2^k \cdot 3^l)$-gon, namely

$$\frac{360^\circ}{n \cdot 2^k \cdot 3^l} = \frac{360^\circ}{n} \cdot \frac{1}{2^k} \cdot \frac{1}{3^l},$$

results from $\alpha_n$ by successive operations of bisection and trisection.

In chapter 5, we have already taken a look at methods of folding regular $n$-gons with $n = 2^k \cdot 3^l$ and $k \geq 1$, but what about the case $n = 3^l$? For $l = 1$, this is simply the equilateral triangle, which is quite straight forward to fold. One method of folding $3^l$-gons is therefore given by first folding an equilateral triangle and then trisecting the angle subtended over its side at the center $l - 1$ successive times. This is essentially the method presented here for the nonagon, that is, the case $l = 2$. (An interesting variant of this idea is presented in [40].)

As illustrated in Figure 8.1 on the following page, the angle subtended over the side of an equilateral triangle at its center is equal to

$$\alpha_3 = \frac{360^\circ}{3} = 120^\circ.$$

The corresponding angle for the nonagon is equal to

$$\alpha_9 = \frac{360^\circ}{9} = 40^\circ = \frac{\alpha_3}{3},$$

and folding the regular nonagon therefore involves the trisection of 120°. As was shown in section 10 (page 33), angle trisection involves the irreducible cubic equation

$$x^3 - \frac{3}{4} \cdot x - \frac{1}{4} \cdot \cos 3\alpha = 0,$$