How to fold simple shapes from A4 paper

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18 February 2012

Contents

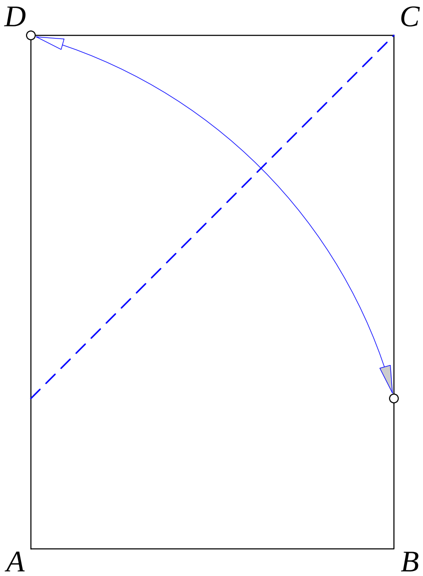
Introduction

Starting with a single sheet of paper, how do you fold shapes like an equilateral triangle or a regular hexagon? On the following pages instructions are given for folding these and other simple shapes.

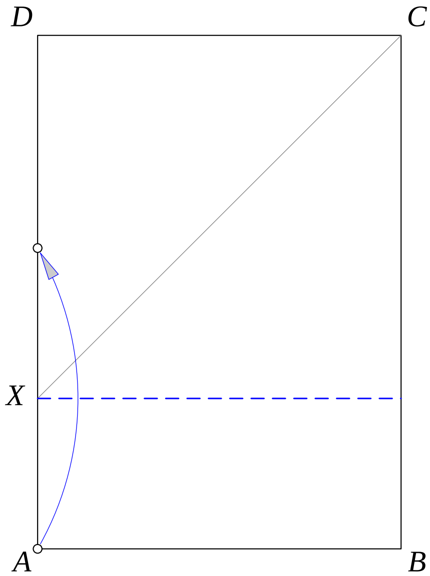
The ability to fold simple shapes quickly provides a ready supply of shapes to handle. This is particularly useful at the experimental stage of geometry, when the names and properties of shapes, and the meanings of words like ‘diagonal’, are being learnt.

Square

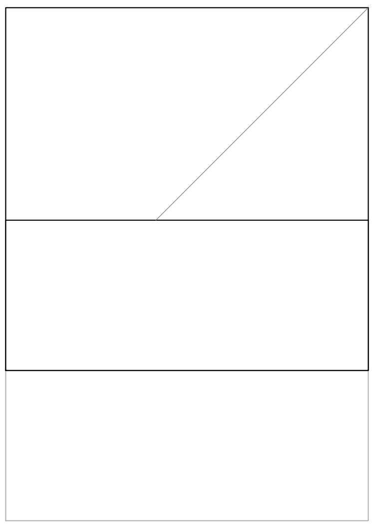
The method is illustrated for A4 paper, but also works for other paper sizes.



STEP 1
Fold D onto BC so that the crease passes through C and unfold.



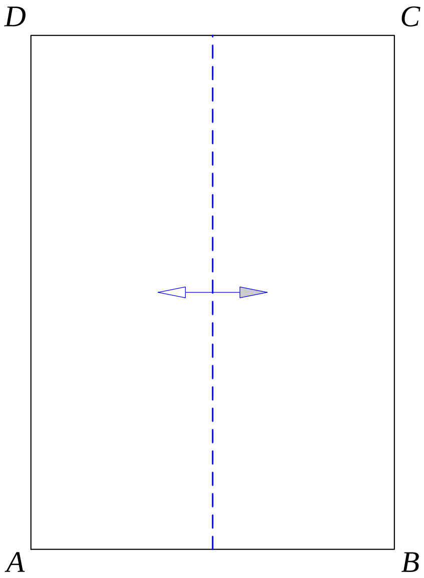
STEP 2
Fold A onto AD so that the crease passes through X.



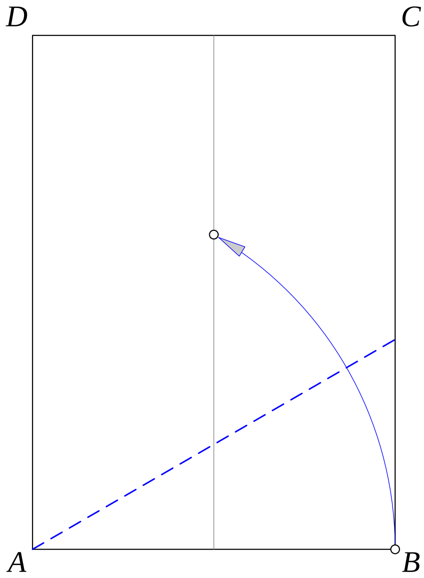
The completed square.

Equilateral triangle

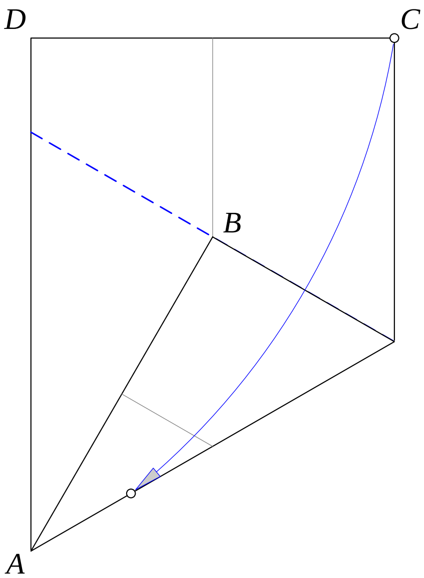
The method is illustrated for A4 paper, but also works for other paper sizes.



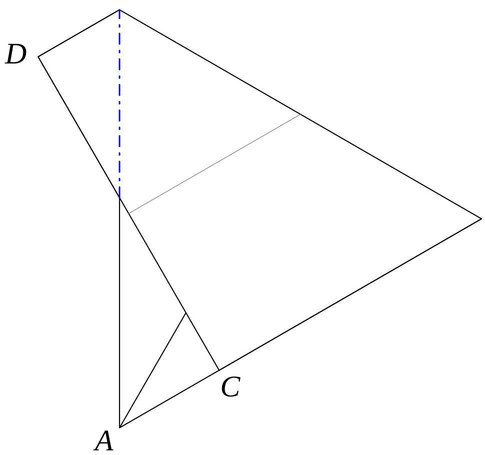
STEP 1
Fold in half lengthwise and unfold.



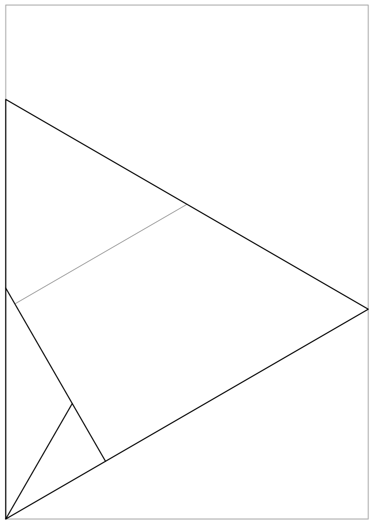
STEP 2
Fold B onto the crease from Step 1 so that the new crease passes through A.



STEP 3
Fold C onto the crease from Step 2 so that the new crease passes through B.



STEP 4
Fold back the flap at D.



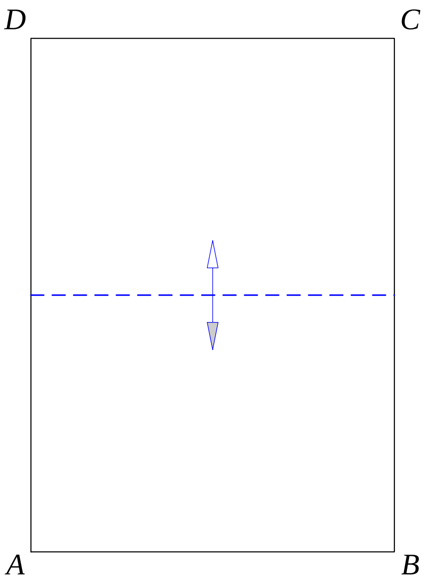
The completed equilateral triangle.

Note

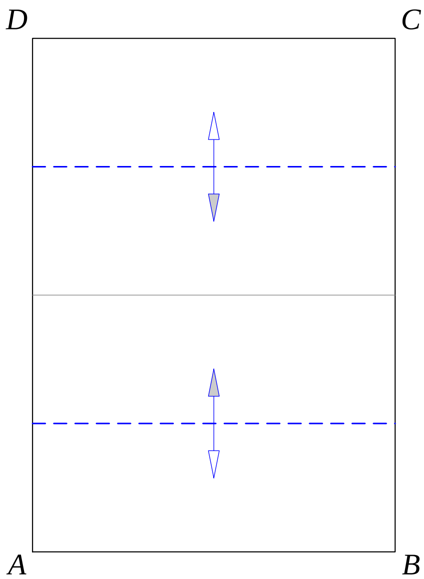
This is not the largest equilateral triangle that can be folded. Instructions for the maximal equilateral triangle for square paper appear in Robert Gereschläger's book [1]; for non-square paper, the method may still be used—just fold the paper into a square first.

Rhombus

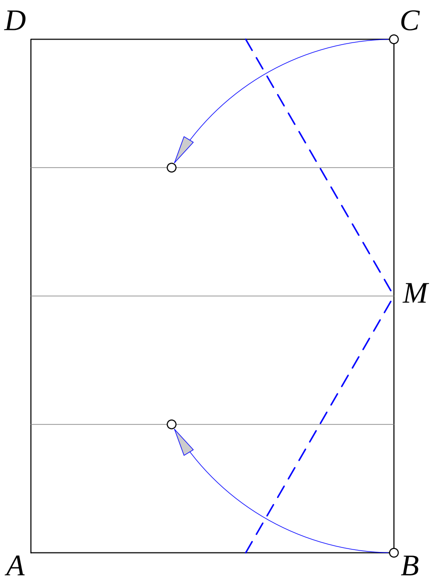
The method is illustrated for A4 paper, but also works for other paper sizes.



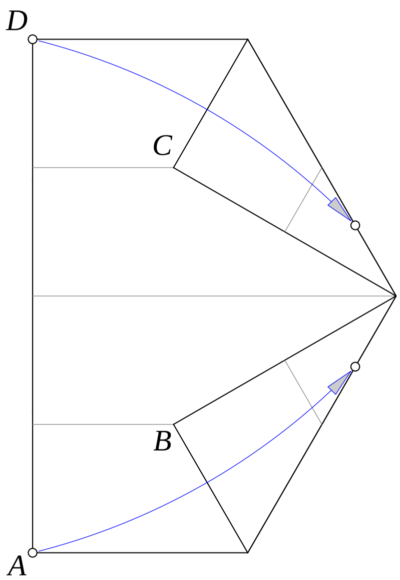
STEP 1
Fold in half widthwise and unfold.



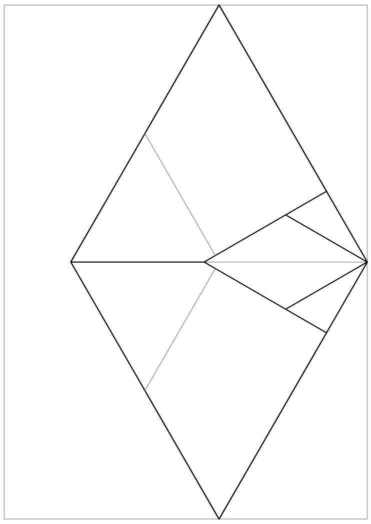
STEP 2
Fold AB and CD onto the crease from Step 1 and unfold.



STEP 3
Fold B onto the lower crease from Step 2 so that the new crease passes through M; repeat with C and the upper crease.



STEP 4
Fold A onto the lower crease from Step 3 so that the new crease passes through B; repeat with D, the upper crease and C.



The completed rhombus.

Note

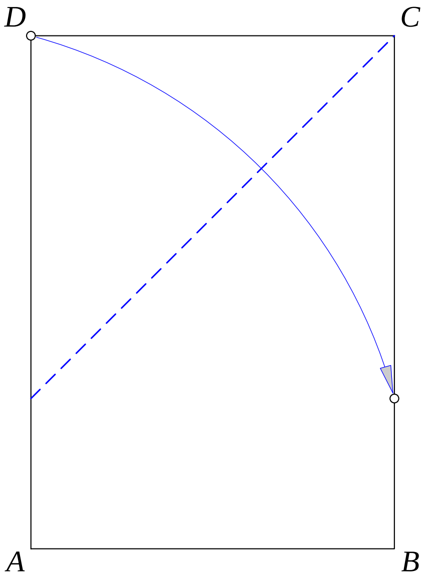
The smaller angles of the rhombus are 60°. Folding the rhombus in half along the (partial) crease from Step 1, produces an equilateral triangle.

Regular hexagon

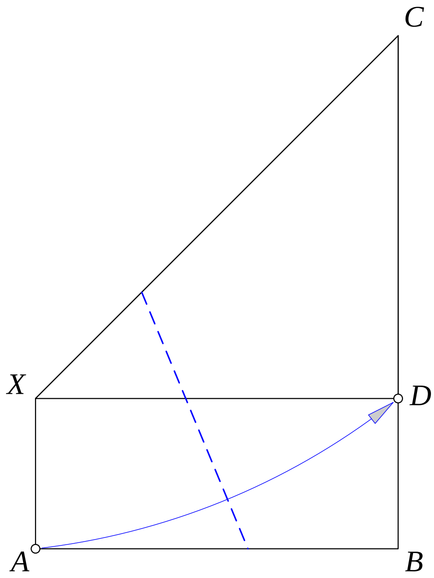
A regular hexagon may be obtained from the rhombus above by folding the top and bottom points to the centre along the (partial) creases from Step 2.

The hexagon obtained in this way is rather small. Instructions for the maximal regular hexagon for square paper appear in Robert Gereschläger's book [1]; for non-square paper, the method may still be used—just fold the paper into a square first. The book also considers the problem of folding maximal regular polygons in general.

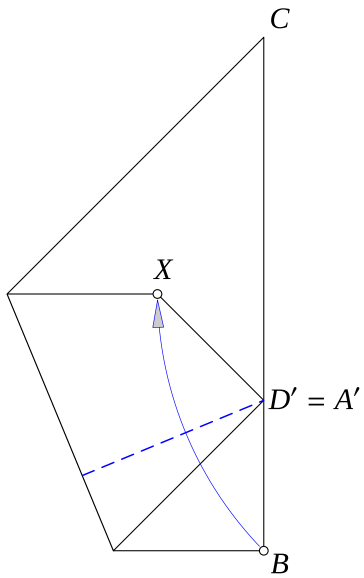
Kite



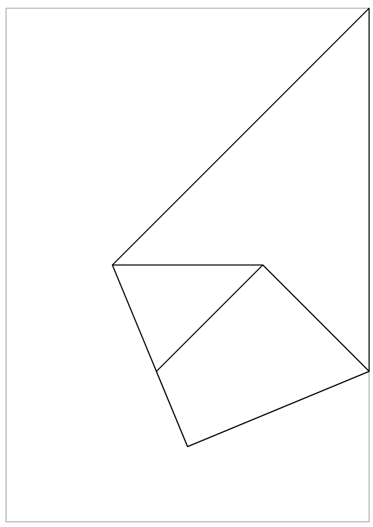
Step 1
Fold D onto BC so that the crease passes through C.



Step 2
Fold A to D.



Step 3
Fold B to X.



The completed kite.

Note

The angles of the kite are $45°$, $112\frac{1}{2}°$, $90°$ and $112\frac{1}{2}°$.

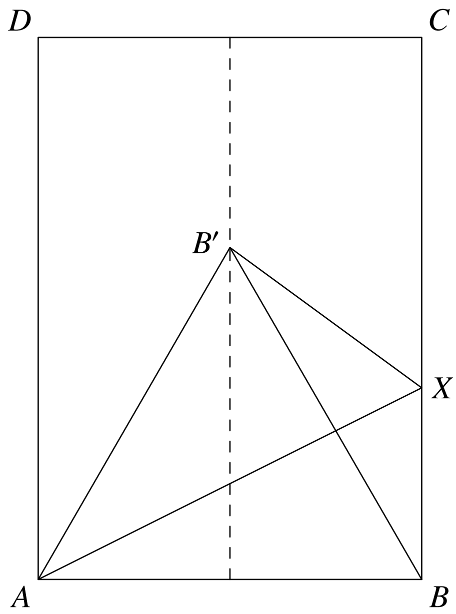
Why do the methods work?

Square

Since $\angle DCX$ is 45° from Step 1, right-angled triangle CDX has equal base angles and therefore $CD = DX$, as required.

Equilateral triangle

Superimposing diagrams of the paper before and after the fold in Step 2, and labelling some of the points, we obtain the following figure, in which AX is the crease from Step 2.



Now $BB' = AB'$ from Step 1, and $AB' = AB$ from Step 2. Therefore ABB' is equilateral and $\angle B'AB = 60°$. Since AX bisects $\angle B'AB$ by construction, it follows that all three angles at A are 30° and hence one angle of the completed triangle at A is 60°.

Finally, the angle of the completed triangle at X is also 60°, because $\angle AB'X = \angle ABX = 90°$ and we know that $\angle B'AX = 30°$. As a result the completed triangle has two angles of 60° and it is therefore an equilateral triangle.

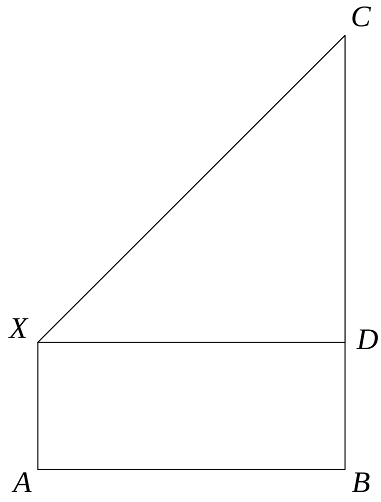
Rhombus

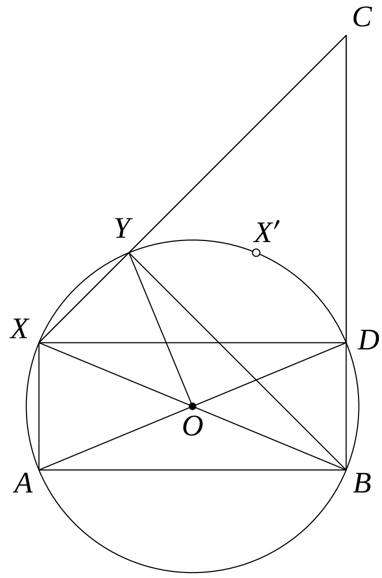
The method is just that for the equilateral triangle applied twice, once to the bottom half of the paper, and once to the top half.

Kite

The method relies on the fact that A4 paper has sides in the ratio $\sqrt{2} : 1$.

Consider the left-hand diagram below, which shows the paper after Step 1. It is clear that $ABDX$ is a rectangle. The right-hand diagram also includes the circle circumscribing the rectangle, whose centre O is the midpoint of the diagonals. Let Y be the point where this circle meets CX. We claim that $CDOY$ is a kite with the required properties. First we shall establish this claim, then show that the remaining folding steps do indeed produce this particular kite.





Firstly, $DO = OY$ because they are both radii.

It follows from Step 1 that $\angle DCX$ is 45°. Also $\angle BYX = 90°$ since BX is a diameter of the circle. Hence $\angle CBY = 45°$, from the angle sum of triangle BCX. Considering chord DY, we deduce that $\angle DOY = 2 \times \angle DBY = 90°$ since the angle at the centre is twice the angle at the circumference.

We now use the fact that A4 paper has sides in the ratio $\sqrt{2} : 1$. Let $AB = 1$, so that $CD = 1$. Also, triangle CBY is a right-angled isosceles triangle (we showed in the last paragraph that its angles are 45°, 45° and 90°) and $BC = \sqrt{2}$. So by Pythagoras' theorem we know that $CY = 1$. Hence $CD = CY$.

We have therefore shown that $CDOY$ is a kite, with $\angle DOY = 90°$.

Next, we know that Step 2 folds A onto D and so the resulting crease lies along OY, since O is the midpoint of AD and OY is perpendicular to AD.

Let X' be the position of X after the fold in Step 2. Then X' also lies on the circle, with XX' perpendicular to OY and therefore parallel to AD. Since BX is a diameter, $\angle BX'X = 90°$, that is, BX' is perpendicular to XX' and therefore BX' is perpendicular to the diameter AD. It follows that when B is folded onto X' then the resulting crease is OD.

We have therefore shown that $CDOY$ is the shape which results from all the folding steps, as required.

References

[1] Robert Geretschläger. *Geometric Origami*. Arbelos, 2008. ISBN: 978-0-9555477-1-3. URL: http://www.arbelos.co.uk/GeometricOrigami.html.