Newton's proof of Kepler's second law

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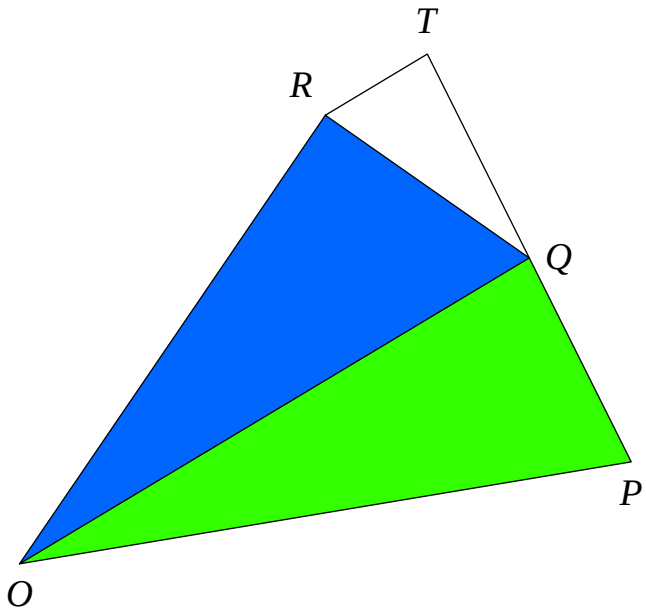
Kepler's second law *A line joining a planet and the sun sweeps out equal areas during equal intervals of time.*

PROOF (NEWTON) Suppose that in equal time intervals a particle moves from P to Q to R under a central force directed towards O; then the acceleration at Q is in direction QO.



Choose the unit of time so that $\overrightarrow{PQ}$ represents the velocity from P to Q. Then $\overrightarrow{QR}$ represents the velocity from Q to R.

Construct $\overrightarrow{QT} = \overrightarrow{PQ}$. Then triangle QTR is the vector triangle relating the velocities and acceleration at Q, with $\overrightarrow{TR}$ representing the acceleration. Hence TR is parallel to QO.

Considering areas of triangles,

$$\begin{aligned} \triangle PQO &= \triangle QTO \quad \text{(equal base, same height)} \\ &= \triangle QRO \quad \text{(same base, equal height)} \end{aligned}$$

and hence equal areas are swept out in equal times. ∎