Newton’s proof of Kepler’s second law

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Kepler’s second law A line joining a planet and the sun sweeps out equal areas during equal intervals of time.

Proof (Newton) Suppose that in equal time intervals a particle moves from $P$ to $Q$ to $R$ under a central force directed towards $O$; then the acceleration at $Q$ is in direction $QO$.

Choose the unit of time so that $\overrightarrow{PQ}$ represents the velocity from $P$ to $Q$. Then $\overrightarrow{QR}$ represents the velocity from $Q$ to $R$.

Construct $\overrightarrow{QT} = \overrightarrow{PQ}$. Then triangle $QTR$ is the vector triangle relating the velocities and acceleration at $Q$, with $\overrightarrow{TR}$ representing the acceleration. Hence $TR$ is parallel to $QO$.

Considering areas of triangles,

\[ \triangle P Q O = \triangle Q T O \text{ (equal base, same height)} \]
\[ = \triangle Q R O \text{ (same base, equal height)} \]

and hence equal areas are swept out in equal times.

\[ \blacksquare \]