

Newton's proof of Kepler's second law

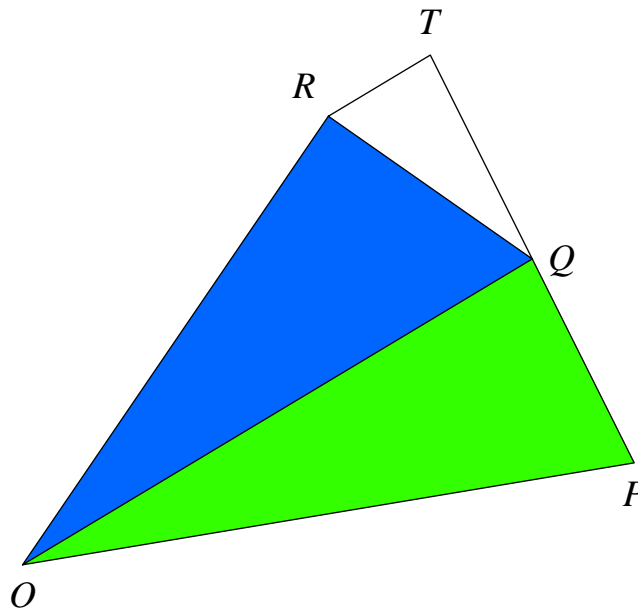
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Kepler's second law *A line joining a planet and the sun sweeps out equal areas during equal intervals of time.*

PROOF (NEWTON) Suppose that in equal time intervals a particle moves from P to Q to R under a central force directed towards O ; then the acceleration at Q is in direction QO .



Choose the unit of time so that \overrightarrow{PQ} represents the velocity from P to Q . Then \overrightarrow{QR} represents the velocity from Q to R .

Construct $\overrightarrow{QT} = \overrightarrow{PQ}$. Then triangle QTR is the vector triangle relating the velocities and acceleration at Q , with \overrightarrow{TR} representing the acceleration. Hence TR is parallel to QO .

Considering areas of triangles,

$$\begin{aligned}\Delta PQQ &= \Delta QTO \quad (\text{equal base, same height}) \\ &= \Delta QRO \quad (\text{same base, equal height})\end{aligned}$$

and hence equal areas are swept out in equal times. ■