Times of flight in a resisting medium

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Suppose that a particle is projected vertically upwards in a resisting medium, so that it ascends and then descends to the starting position. Is the time of ascent greater than, equal to, or less than the time of descent? Does the answer depend on the nature of the resistive force?

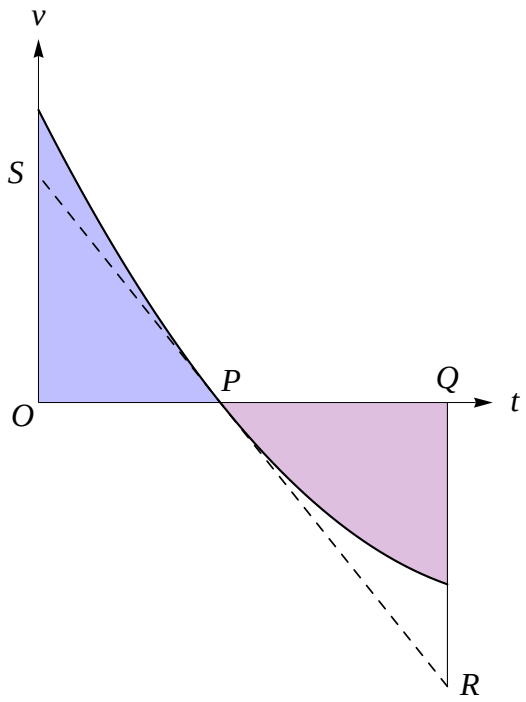
Result *Whatever the resistive force, the time of ascent is less than the time of descent.* □

If v is the velocity at time t, then we have

$$\frac{dv}{dt} < -g \quad \text{for upward motion}$$

$$\text{and} \quad \frac{dv}{dt} > -g \quad \text{for downward motion.}$$

Hence we have a velocity-time graph of the form shown in the figure, where P corresponds to the top of the motion, the line SR has slope $-g$, and OP and PQ are the times of ascent and descent, respectively.



Of course, the precise shape of the curve will depend on the nature of the resistance; the curve may also have discontinuous gradient at P. However, what follows is not affected by such differences.

Now the distances up and down are the same so that the blue and purple shaded areas are equal.

Hence area $\triangle OPS$ < area $\triangle PQR$ and it follows that $OP < PQ$, so that the time of ascent is less than the time of descent. ■