Lune
Triangle
Areas I
Areas II
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which means that

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Let the spherical triangle have angles \( r \), \( g \) and \( b \). From the results above we get

\[
T = (r + g + b - \pi)R^2,
\]

a result sometimes called \textit{Girard’s theorem}.

The quantity \( r + g + b - \pi \) is the \textit{spherical excess} of the triangle.