Three squares

ANDREW JOBBINGS
www.arbelos.co.uk

16 September 2016

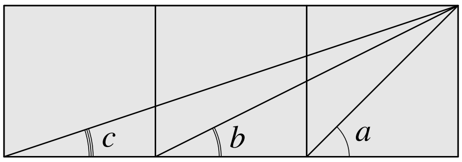

Figure 1

A classic problem is that $a = b + c$, where the angles a, b and c are angles in the three equal squares shown in figure 1.

This is equivalent to $\arctan 1 = \arctan \frac{1}{2} + \arctan \frac{1}{3}$, which may be proved by making use of the formula for $\tan(A + B)$. However, a more straightforward approach is as follows.

Figure 2 is figure 1 with some more squares added, and diagonals of two 2×1 rectangles and one 3×1 rectangle drawn. The angles indicated are equal in pairs because each is the angle between a long side and a diagonal of either a 2×1 or a 3×1 rectangle.

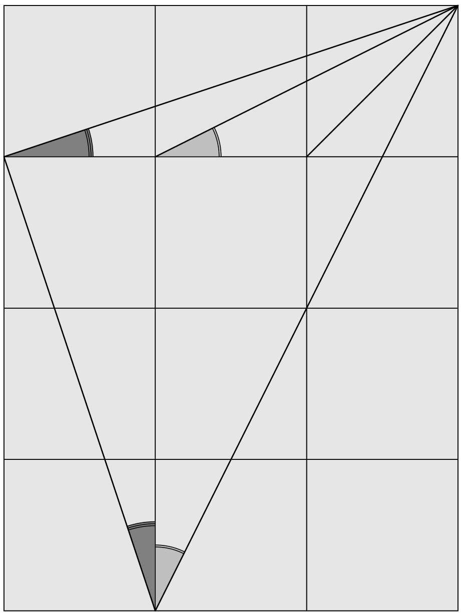

Figure 2

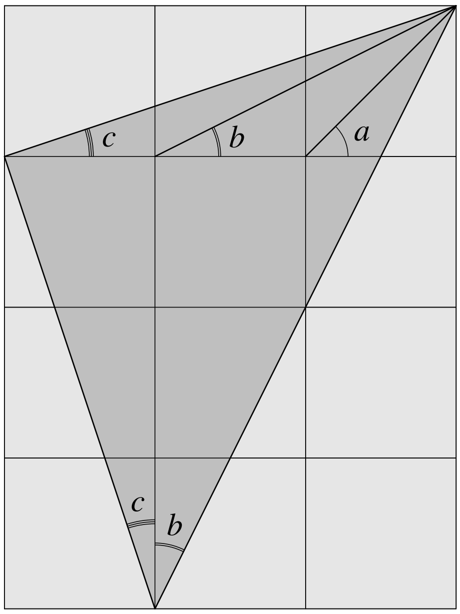

Figure 3

In figure 3, the dark triangle is right-angled and isosceles, because the two legs are diagonals of perpendicular 3×1 rectangles. Hence $b + c = 45°$.

Now a is the angle between a side and a diagonal of a square, so that $a = 45°$. It follows that $b + c = a$.