Book Review by Karen Birnie

Progress to Higher Mathematics, by Mary Teresa Fyfe, Andrew Jobbings and Kitty Kilday, Arbelos 2007.

It is a common belief amongst those involved in mathematical education in Scotland that many students lack confidence and ability at Higher level due to a lack of mathematical, and in particular algebraic, skills. *Progress to Higher Mathematics* has been written as a direct response to address these concerns regarding the difficulties faced by students progressing from Standard Grade Credit or Intermediate 2 to Higher Mathematics.

The authors suggest that *Progress to Higher Mathematics* may be used either as a precursor to the Higher syllabus or as introductory exercises throughout the teaching of the course. They claim the exercises in the book will:

- build on previous knowledge and skills gained at Standard Grade or Intermediate 2
- consolidate the essential skills necessary for Higher
- enhance algebraic expertise
- utilize graphicacy as a means of improving mathematical understanding
- develop the geometrical ideas needed for a through understanding of analytical geometry.

Progress to Higher Mathematics comprises nine chapters covering the essential skills in algebra and graphicacy required for Higher Mathematics. Within each chapter, there are several exercises, each with clearly defined learning objectives. The opportunity for extension towards Higher is available through many of the highlighted exercises. Non-calculator skills are also reinforced through specified non-calculator exercises.

The first two chapters, *Preliminaries* and *Solving Equations*, mainly consolidate the algebraic foundations required from Standard Grade Credit or Intermediate 2, such as expanding and factorising expressions, solving equations (linear, simultaneous, with fractions, quadratics and cubics), surds and indices. These skills are then linked together to form extension questions, such as "Solve the equation $4(d + 2)^2 = 24$, leaving your answer in surd form".

The third chapter, *Lines and Circles*, extends students' knowledge from gradient and equation of a straight line to considering the properties of parallel lines, midpoints, collinearity, the relationship between angles and gradients and the diameter and tangent properties of a circle. If students are secure in this knowledge, they will find the concepts introduced in the straight line, and circle much more accessible.

Chapters four and five investigate parabolic and cubic curves through *Curve Sketching and Equation of Curves*. This skill, of being able to visualise a curve given its equation and vice versa, is invaluable for so many Higher topics, but is one of the skills that many pre-Higher students lack confidence and experience in. The technique of being able to change the equation of a parabola from the form $y = k(x – a)(x – b)$ to the form $y = k(x – p)^2 + q$ in order to sketch the parabola more easily is also addressed.

The sixth chapter, *Intersecting Lines and Curves*, reinforces the intersection of two straight lines before moving on to consider the intersection between a straight line and a parabola, a straight line and a cubic and two parabolas. These skills are particularly useful for finding the limits of integration required when calculating the area between two curves.

Chapters seven and eight, *Trigonometric Graphs* and *Trigonometry*, reinforce and extend students' knowledge of trigonometry. Several of the exercises in these chapters are non-calculator and require students to know and use exact trigonometric values. The exercises in these chapters provide practice in finding the maximum and minimum values of a graph and points of intersection between the graph and a straight line (often the axes) for progressively more complicated trigonometric graphs. There is also further practice in the use of the trigonometric identities ($\sin^2 x + \cos^2 x \equiv 1$ and $\tan x \equiv \frac{\sin x}{\cos x}$), calculating exact values for angles greater than 90° and calculating the values of other trigonometric ratios from a given ratio.

The final chapter, *Algebra*, covers functions, constructing expressions and further surds and indices. Higher students often struggle with optimization questions which start with the construction of an expression, particularly involving volume and/or surface area. If students are able to construct the expression involved themselves, they will then tackle the optimization part of the question with more confidence and understanding. Similarly, question due to an inability to arrange the expression into index form, from which they can then differentiate or integrate. The exercise on further surds and indices provides the opportunity to gain experience at writing quite complicated expressions such as $\frac{x^2 - 4x}{\sqrt{x}}$ as a sum of separate terms, giving the answer in index many students are unable to tackle a simple calculus form. Students who can deal confidently with the manipulation of such expressions will then be able to concentrate more fully on the new concepts involved in calculus.

This book offers students the opportunity to do exactly as it says in its title, *Progress to Higher Mathematics*!